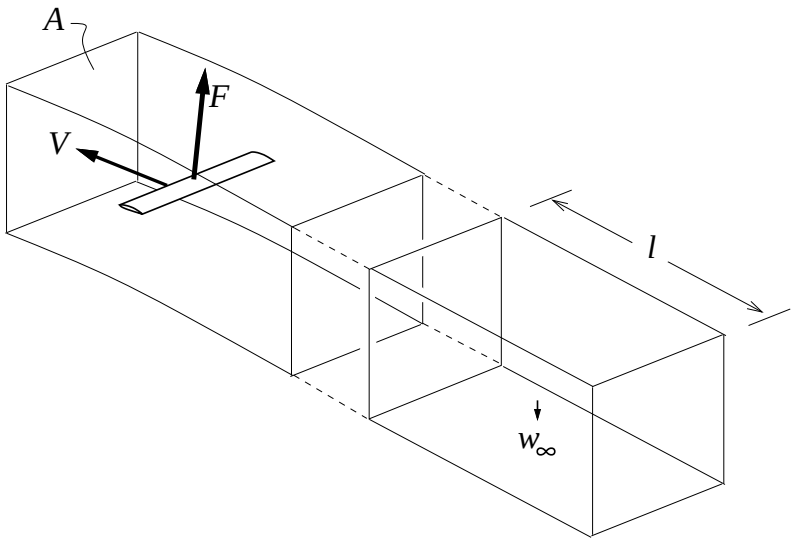
Unified Engineering Spring 2008
Fluids Problem 5

A somewhat crude but simple model of a lifting wing flying at speed V in air of density ρ uses the assumption that the wing pushes down on a streamtube of air of some effective cross-sectional area A. The total force F has components L and D_i perpendicular and parallel to V.



Consider an ℓ-long slug of this streamtube of air, which is imparted with vertical velocity w_∞ by the wing passing through it.

a) Determine the slug's mass, and momentum relative to the undisturbed atmosphere. Relate this to the lift L of the wing. Hint: Consider the impulse of L on the slug.

b) Determine the slug's kinetic energy relative to the undisturbed atmosphere. Using conservation of energy arguments, relate this to the induced drag D_i of the wing.

c) In the notes and in class, the induced drag was interpreted as the aft component of F, which is tilted by a downwash velocity w at the wing. Comparing this model with a) and b), relate w at the wing to the w_∞ far behind the wing.

d) The wing's speed is now increased to $V' = 2V$, while the lift remains constant, e.g. equal to the airplane weight. Determine how w_∞ and D_i change according to the model in a), b).